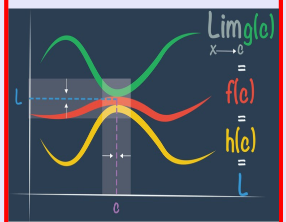
Math 261

Fall 2022

Lecture 16



Calculus I | **Name:** ______________________

Class Quiz 4 | **Signature:** ______________________

No Work ⇔ No Points

Be Neat , Organized & Show Detailed Work

1. (6 points) Evaluate: $\lim\limits_{x\to-\infty} \dfrac{2x}{\sqrt{25x^2+10x}}$ $= \dfrac{-\infty}{\infty}$ I.F. as $x \to -\infty$

$x = -\sqrt{x^2}$

$$\lim_{x\to-\infty} \frac{\frac{2x}{x}}{\frac{\sqrt{25x^2+10x}}{x}} = \lim_{x\to-\infty} \frac{2}{-\sqrt{\frac{25x^2}{x^2}+\frac{10x}{x^2}}} = \lim_{x\to\infty} \frac{2}{-\sqrt{25+\cancel{\frac{10}{x}}}} \to 0 = \frac{2}{-5}$$

1. $-2/5$

2. (6 points) For $\epsilon = 0.2$, find $\delta > 0$ such that $\lim\limits_{x\to 0} \sqrt[3]{x} = 0$. ✓

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|\sqrt[3]{x} - 0| < .2$ " $|x - 0| < \delta$

Cube both sides

$|\sqrt[3]{x}| < .2$ whenever $|x| < \delta$ → $|x| < .2^3$

$|x| < .008$

2. $\delta = .008$

3. (8 points) Find the first derivative of the function $f(x) = x^2 - 4x + 3$ by using the definition of derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 4(x+h) \cancel{+3} - x^2 + 4x \cancel{-3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-4x} - 4h \cancel{-x^2} \cancel{+4x}}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h-4)}{\cancel{h}} = \lim_{h\to 0}(2x + \cancel{h} - 4) \quad h \to 0$$

3. $\boxed{f'(x) = 2x-4}$

Page 1 of 1 | Class Quiz 4 | Total Points: 20

For function $f(x)$

first derivative $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

New notation:

$$f'(x) = \frac{d}{dx}[f(x)]$$

If $y = f(x)$, then

$$y' = f'(x) = \frac{dy}{dx}$$

Find $f'(x)$ for $f(x) = x^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

Recall

$A^3 - B^3 =$

$(A-B)(A^2 + AB + B^2)$

$$= \lim_{h \to 0} \frac{(\cancel{x}+\cancel{h} - \cancel{x})((x+h)^2 + (x+h)x + x^2)}{\cancel{h}}$$

$$= \lim_{h \to 0} [(x+\cancel{h})^2 + (x+\cancel{h})x + x^2]$$

$$= x^2 + x \cdot x + x^2 = \boxed{3x^2}$$

Find the first derivative of any constant function.

$f(x) = c$

$f'(x) = 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} \frac{0}{h}$$

$$= \lim_{h \to 0} 0 = \boxed{0}$$

Find the first derivative of any linear function

$f(x) = mx + b$

$f'(x) = m$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{m(x+h) \cancel{+b} - mx \cancel{-b}}{h} = \lim_{h \to 0} \frac{\cancel{mx} + m\cancel{h} - \cancel{mx}}{\cancel{h}}$$

$$= \lim_{h \to 0} m = \boxed{m}$$

Find the x-coordinate of the point on any quadratic function with horizontal tangent line.

$f(x) = ax^2 + bx + c$

$a \neq 0$

has zero slope

$f'(x) = 0$

1) find $f'(x)$

2) Solve $f'(x) = 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{a(x+h)^2 + b(x+h) \cancel{+c} - ax^2 - bx \cancel{-c}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{ax^2} + 2axh + ah^2 \cancel{+bx} + bh - \cancel{ax^2} \cancel{-bx}}{h}$$

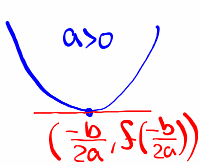
$$= \lim_{h \to 0} \frac{\cancel{h}(2ax + ah + b)}{\cancel{h}} = \lim_{h \to 0} [2ax + \cancel{ah}^{\to 0} + b]$$

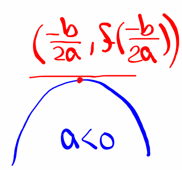
$$= 2ax + b$$

Now Solving $f'(x) = 0$

$2ax + b = 0$

$\boxed{x = \frac{-b}{2a}}$





Find a pattern or formula for first derivative of $F(x) = f(x) \cdot g(x)$.

$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{g(x+h)[f(x+h) - f(x)] + f(x)[g(x+h) - g(x)]}{h}$$

$$= \lim_{h \to 0} \frac{g(x+\overset{0}{h})[f(x+h) - f(x)]}{h} + \lim_{h \to 0} \frac{f(x)[g(x+h) - g(x)]}{h}$$

$$= g(x) \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + f(x) \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= g(x) \cdot f'(x) + f(x) \cdot g'(x)$$

$$\boxed{\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)}$$

Product Rule

Recall from last week

$f(x) = \sin x$ and $f(x) = \cos x$

$f'(x) = \cos x$ and $f'(x) = -\sin x$

So find $\frac{d}{dx}[x^3 \sin x] =$

$f(x)$ $g(x)$

Product Rule

$$\frac{d}{dx}[x^3] \cdot \sin x + x^3 \cdot \frac{d}{dx}[\sin x]$$

$$f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$= \boxed{3x^2 \cdot \sin x + x^3 \cdot \cos x}$$

Find $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x+h)} + \frac{f(x)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{f(x+h) - f(x)}{g(x+h)} + f(x)\left[\frac{1}{g(x+h)} - \frac{1}{g(x)}\right]}{h}$$

LCD $= g(x+h)\cdot g(x)$

$$= \lim_{h\to 0} \frac{1}{g(x+h)} \cdot \frac{f(x+h)-f(x)}{h} + f(x) \lim_{h\to 0} \frac{\left[\frac{1}{g(x+h)} - \frac{1}{g(x)}\right]}{h}$$

$$= \frac{1}{g(x)} \cdot f'(x) + f(x) \cdot \lim_{h\to 0} \frac{g(x) - g(x+h)}{h\, g(x+h)\, g(x)}$$

$$= \frac{f'(x)}{g(x)} + f(x) \cdot - \lim_{h\to 0} \frac{g(x+h) - g(x)}{h\, g(x+h)\, g(x)}$$

$$= \frac{f'(x)}{g(x)} - f(x) \cdot \frac{1}{[g(x)]^2} \cdot \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \frac{f'(x)}{g(x)} - \frac{f(x)}{[g(x)]^2} \cdot g'(x) = \frac{f'(x) \cdot g(x)}{[g(x)]^2} - \frac{f(x)g'(x)}{[g(x)]^2}$$

$$= \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$ Quotient Rule

---

$f(x)$

$$\frac{d}{dx}\left[\frac{x^2}{\cos x}\right] = \frac{\frac{d}{dx}[x^2] \cdot \cos x - x^2 \cdot \frac{d}{dx}[\cos x]}{[\cos x]^2}$$

$g(x)$

$$= \frac{2x \cdot \cos x - x^2 \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{2x \cos x + x^2 \sin x}{\cos^2 x}$$

find $\frac{d}{dx}\left[\tan x\right] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$

$$= \frac{\frac{d}{dx}[\sin x] \cdot \cos x - \sin x \cdot \frac{d}{dx}[\cos x]}{\cos^2 x}$$

$$= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \boxed{\sec^2 x}$$

$$\boxed{\frac{d}{dx}\left[\tan x\right] = \sec^2 x}$$